PERGAMON

International Journal of Solids and Structures 38 (2001) 1737–1748

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

A case study on frequency response optimization

Sergio Butkewitsch *, Valder Steffen Jr.

Dynamic Systems Laboratory, School of Mechanical Engineering, Federal University of Uberlândia, Campus Santa Monica, 38400-902, Uberlândia, MG, Brazil

Received 1 September 1999; in revised form 15 January 2000

Abstract

This paper presents an application of structural synthesis techniques to the design of an exhaust system belonging to a heavy truck. The system is excited by the vehicle's powertrain over a wide frequency range, containing five resonant peaks. Through the optimization procedure, the finite element method model is modified, and the profile of the supporting beams is automatically redesigned to achieve superior performance: first, obtaining optimum values for the natural frequencies and later minimizing the dynamic displacements. It is important to mention that structural synthesis software as the one employed in this study (VMA/GENESIS version 3.0) usually cannot deal with entities that present different values at the various analysis steps as is the case of frequency dependent displacement. It means that such physical terms cannot be computationally expressed as objective functions or constraints for optimization purposes. This problem is overcome by means of a mathematical artifice, the "Beta Method". This scheme relies on an auxiliary design variable for optimization and minimizes the maximum value assumed by a certain entity. The rules for expressing the optimization problem better in terms of the "Beta" variable will also be outlined.

Keywords: Automated structural synthesis; Frequency response; Optimization

1. Introduction

Structural synthesis techniques result from the combination of non-linear optimization procedures and analysis modules in order to achieve a high level of automation in design engineering practice (Vanderplaats, 1998; VMA Engineering, 1996; Moore, 1992). By means of an iterative process, the numerical optimizer is able to search the best possible configuration within a set of design spaces, which are possible mathematical representations of the engineering problem under consideration. As part of the design space definition, the engineer is required to present an initial design configuration and the criteria to be fulfilled by the resulting one.

Thus, mathematically speaking, the design optimization task is defined as a function whose extreme (usually minimum) value is to be determined:

* Corresponding author.
E-mail addresses: sbut@mecanica.ufu.br (S. Butkewitsch), vsteffen@mecanica.ufu.br (V. Steffen Jr.).

0020-7683/01/$ - see front matter
PII: S0020-7683(00)00133-5

$$\min\left[F\left(\underset{\sim}{\mathbf{X}}\right)\right]. \tag{1}$$

The equation above defines the *objective function*, dependent of an *n*-dimensional vector $\underset{\sim}{\mathbf{X}}$, which contains the parameters to be modified in order to achieve the design goal. These parameters are usually called *design variables* or *decision variables*.

In a realistic environment, however, attention must be dispensed to a very important detail: the pursuit for the design goal is always limited by certain constraints. These constraints can be of several nature, such as physical, economical and also due to production aspects and time requirements. Despite their different motivations, the constraints share a common feature: as well as the objective function, they also depend upon the design variables stored in vector $\underset{\sim}{\mathbf{X}}$. The design variables are updated at each iteration in order to find a better value for the objective function, and the constraint functions assume new values as well.

The constraint functions are classified into two groups:

$$\begin{aligned}&\text{Inequality constraints}\quad G_j\left(\underset{\sim}{\mathbf{X}}\right)\leqslant 0,\\&\text{Equality constraints}\quad H_{k(\underset{\sim}{\mathbf{X}})}=0.\end{aligned} \tag{2}$$

Another category of constraints consists of boundaries imposed over the values of the design variables, which are usually called *side constraints*:

$$\underset{\sim}{\mathbf{X}}^{\text{LOWER}}\leqslant\underset{\sim}{\mathbf{X}}\leqslant\underset{\sim}{\mathbf{X}}^{\text{UPPER}}. \tag{3}$$

Once the optimization problem is formulated in mathematical terms, one can choose among several solution methods in order to obtain the optimal configuration for the system being automatically designed. Several classifications of these methods have been carried out by many authors in an attempt to define which group of algorithms, according to its operational features, is best suited for each category of practical problems. The most popular and widely used methods are calculus based methods, also subdivided into zero, first and second order methods according to the order of the derivatives evaluated in order to define search directions used along the iterations performed to find the optimum.

Another classification divides the calculus based methods in sequential and direct methods. For the problems to be presented in this paper, a direct method known as "the modified method of feasible directions" (MMFD) was chosen due to its superior behaviour in the presence of a large amount of constraints (a feature it shares with the majority of direct methods) as is usually the case in engineering applications. The MMFD algorithm can be briefly presented as follows:

$$\min\left(\underset{\sim}{\nabla}f(x)\cdot\underset{\sim}{\mathbf{S}}\right), \tag{4}$$

where $\underset{\sim}{\nabla}f(x)$ is the gradient vector of the objective function in the current design, and $\underset{\sim}{\mathbf{S}}$ is the vector expressing the search direction in the design space. The lower the internal product result in Eq. (4), the smaller the angle formed between the search direction and the objective function gradient vector, meaning that the navigation throughout the design space occurs as close as possible to the steepest ascent/descent direction. Also, in order to keep the constraints behind violation and to limit the magnitude of the $\underset{\sim}{\mathbf{S}}$ vector, the following constraint equations are considered:

$$\underset{\sim}{\nabla}g(x)\cdot\underset{\sim}{\mathbf{S}}\leqslant 0, \tag{5}$$

$$\underset{\sim}{\mathbf{S}}\cdot\underset{\sim}{\mathbf{S}}\leqslant 1, \tag{6}$$

where $\underset{\sim}{\nabla}g(x)$ stands for the gradient vector of the constraint functions in the current design.

In the special cases where equality constraints apply, the set comprised by Eqs. (5) and (6) has to be completed with the following expression:

$$[\mathbf{B}] \cdot \underset{\sim}{\mathbf{S}} = 0, \tag{7}$$

where the matrix [**B**] contains the gradients of all the equality constraints belonging to the optimization problem.

All the fundamental entities that appear in the formalism depicted above are expressed in mathematical terms, that is, functions whose values need to be repeatedly evaluated as long as the optimization task proceeds. Thus, any software application designed for optimization purposes possesses a module dedicated to function (objective and constraints that represent the structure behaviour) evaluation. In engineering practice, some important features are required from this calculation module: robustness, ability to deal with large/complex problems and discrete operation (since analytical expressions are seldom available). Such characteristics are displayed by the finite element method (FEM), widely used for engineering analysis purposes. Some important details concerning dynamic finite element analysis will be addressed later in this paper since their understanding is essential to the proper formulation of an optimization based design procedure for structural vibrations.

2. Computer feasibility requirements on FEM based structural synthesis

In Section 1, no special consideration was made about the interaction established between the numerical non-linear optimizer and the finite element solver responsible for the analysis module. At first glance, their interaction is depicted in Fig. 1.

However, when one thinks about the CPU effort resulting from repetitive FEM solver calls by the optimizer, it is worth remembering that the solver module depicted in Fig. 1 seldom requires less than 95% of time for a complete optimization. So, in reality, for the sake of computational feasibility, the coupling between the optimization and analysis modules must possess a set of model reduction algorithms that operate based on the hypothesis that some parts of the finite element model offer no contribution for the optimization process itself at certain stages. Fig. 2 describes such an implementation scheme.

The true structural synthesis approach described in Fig. 2 counts with the participation of an approximate problem generation module, which comprises a set of condensation techniques that attempt to mimic the judgement performed by human engineers when they need to elect priorities and disregard system aspects that will not contribute significantly to the design optimization of a structure at a given stage. The algorithms currently available to generate approximate problems are shortly commented in the sequence of

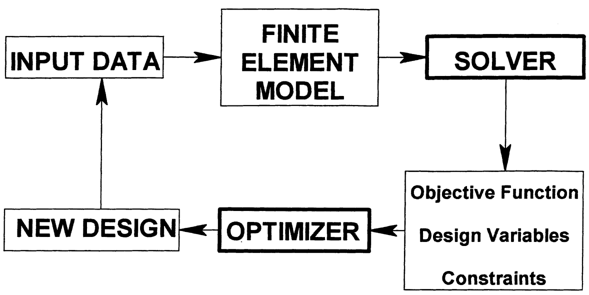

Fig. 1. Optimizer/FEM direct linking block diagram.

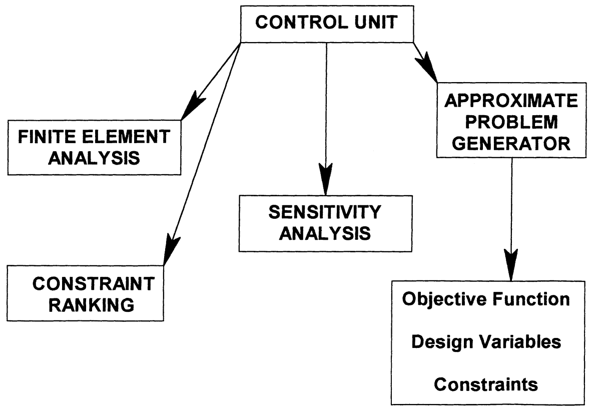

Fig. 2. Structural synthesis approach.

this paper (significant research effort is being deployed within the engineering community in order to further develop condensation methods useful to a broad set of multidisciplinary optimization problems).

2.1. Design variable linking

By establishing linear relations among design variables, one can reduce the amount of independent variables to be evaluated, resulting in a lower demand for CPU power, as shown in Fig. 3.

This technique also displays other two important advantages besides the one mentioned above:

1. The relations among design variables are directly controlled by the user. This helps to keep physical insight about the design.
2. The laws of dependence can be useful to enforce desirable design features, such as symmetry and parallelism (Fig. 3).

2.2. Constraint deletion

If a given subset of all the prescribed constraints has no risk of violation, it is useless to waste CPU time with their evaluation. Hence, for the sake of feasibility, the constraints far from the violation threshold can be neglected until their importance grows (i.e., risk of violation arises) at a different stage of the automated design process. The design engineer can define a numerical threshold TRS, and only the constraints whose normalized values are above TRS will be considered at the moment. Fig. 4 illustrates this concept graphically.

2.3. Constraint screening

Since the FEM model is discrete over the design domain, the various constraints prescribed over a physical response related to a particular group of elements can be replaced by a constraint applied to the same physical quantity, obtained just at the most representative elements (only one in the limiting case) of the group. For example, let it be a group of thousands of shell finite elements employed to model an aeronautic airfoil. If one desires to minimize the structure's weight keeping track of fatigue stress levels, constraints must be prescribed over all the elements of the airplane's wing. Just a few elements, however,

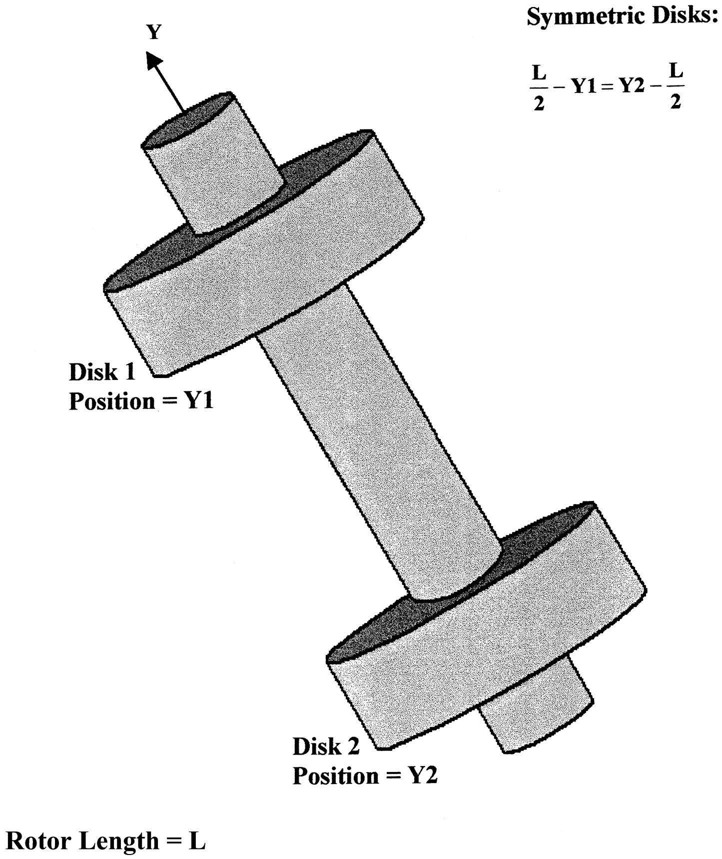

Fig. 3. Illustration of design variable linking.

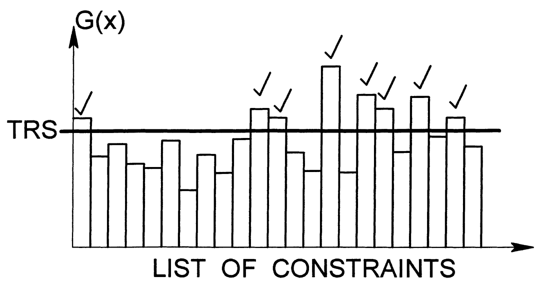

Fig. 4. Constraint deletion.

can be considered at each optimization iteration due to their superior representativity in comparison with the others. This principle, mainly the role of the NSTR (constraints retained per region) parameter, is illustrated in Fig. 5.

It should be noted that the procedures of constraint deletion and screening are overlapped (i.e., used in conjunction) in order to avoid the heavy calculations involved in the evaluation of unnecessary constraint functions. Thus, the number of such functions is reduced to the least possible.

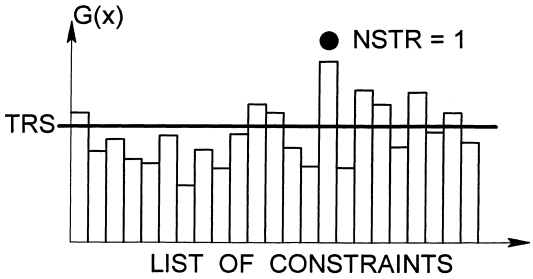

Fig. 5. Constraint screening.

2.4. Formal approximations

Up to this moment, the model reduction techniques presented acted on quantitative basis, that is, only the alternatives created to reduce the number of independent design variables and constraints were indicated.

Although these solutions are effective, more CPU power can be saved if one analyses the qualitative aspects related to the functions evaluated during the optimization procedure. Even if several simple functions are eliminated by means of constraint deletion and screening and a very few complex, highly non-linear functions remain, too much computer effort will be employed. For this reason, whenever possible, it is interesting to simplify the functions involved by means of linearization. This can be performed by expanding the functions in a Taylor Series to be truncated at the first term, as indicated below:

$$
\begin{aligned}
f(x^0+\Delta x) &= f(x^0) + \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x^0}\cdot\Delta x + \left.\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}\right|_{x^0}\cdot\frac{\Delta x^2}{2!} + \left.\frac{d3f}{dx^3}\right|_{x^0}\cdot\frac{\Delta x^3}{3!} + \cdots,\\
\tilde{f}(x^0+\Delta x) &= f(x^0) + \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x^0}\cdot\Delta x. \qquad (8)
\end{aligned}
$$

2.5. Advanced techniques – replacement by a physically equivalent quantity

Still in the effort of simplifying the function calculations involved in the numerical non-linear optimization process, a special group of algorithms was developed with the objective of replacing certain given functions by others which are physically equivalent, but whose calculations require less computational power.

Internal forces instead of stresses (Vanderplaats and Salajegheh, 1989): A very common structural optimization task aims to obtain the minimum possible mass without violating stress constraints. Since the material (and consequently its density) are very seldom considered as design variables, the optimizer has to impose changes to the geometric parameters, and area is chosen, most of the times, as the design variable.

In such a formulation, the objective function displays a linear, explicit relation with respect to the design variables. The same, however, does not hold true for the constraints because stresses and areas relate with each other by means of a reciprocal mathematical function. This situation poses a special difficulty for the optimizer because the optimization problem is usually strongly driven by the constraints, which get involved with this non-linearity problem in this particular kind of formulation.

Indeed, one would prefer, for the sake of computational economy, the objective function to become non-linear, and the constraints linear with respect to the design variables. This switch can be done if the design variables became the reciprocal of the areas, which is equivalent to integrate the stresses with respect to the areas, obtaining the internal forces. For this reason, it is a usual procedure to replace stresses by internal forces in structural synthesis problems.

Rayleigh coefficient instead of eigenvalues (Canfield, 1990): Another very usual optimization challenge in design engineering environment is the achievement of an ideal dynamic behaviour in terms of the structure natural frequencies, avoiding resonances or critical speeds.

The mathematical model used to deal with dynamic systems, however, leads to complicated matrix manipulations used to determine the eigenvalues. Keeping the objective of saving computational effort, the natural frequencies and, in particular, the lowest eigenvalue can be estimated based on the Rayleigh coefficient, a scalar that relates the kinetic and elastic energies of the vibrating structure:

$$\lambda = \omega^2 = \frac{\underset{\sim}{\phi}^T[\mathbf{K}]\underset{\sim}{\phi}}{\underset{\sim}{\phi}^T[\mathbf{M}]\underset{\sim}{\phi}} = \frac{U}{T}. \tag{9}$$

3. Mathematical model and finite element analysis approach (Blakely, 1993)

Linear vibration phenomena are governed by the following second order matrix differential equation,

$$[\mathbf{M}]\underset{\sim}{\ddot{\mathbf{x}}} + [\mathbf{C}]\underset{\sim}{\dot{\mathbf{x}}} + [\mathbf{K}]\underset{\sim}{\mathbf{x}} = \underset{\sim}{\mathbf{F}}(t), \tag{10}$$

which represents a multi degree of freedom system. Eq. (10) can be solved for the homogeneous case ($\{\mathbf{F}(t)\} = 0$), which corresponds to modal analysis by considering the displacement vector $\{x\}$ as given by

$$\underset{\sim}{\mathbf{x}} = \underset{\sim}{\mathbf{X}}e^{i\omega t} \tag{11}$$

leading to

$$\left([\mathbf{K}] + i\omega[\mathbf{C}] - \omega^2[\mathbf{M}]\right)\underset{\sim}{\mathbf{X}} = 0 \Rightarrow \det\left([\mathbf{K}] + i\omega[\mathbf{C}] - \omega^2[\mathbf{M}]\right) = 0, \tag{12}$$

which is the eigenproblem associated to the equation of motion. For each frequency ωi, the components of $\underset{\sim}{\mathbf{X}}$ are normalized to give the correspondent eigenvector or mode shape (some synthesis software allow for eigenvector components optimization, others do not).

Besides modal analysis, the problem presented in this article deals with the steady state responses of structures forced by time-varying loading (indeed, for damped cases, only the steady state solution can be considered for most applications). So, the complete response is obtained as the result of the free response added to the particular solution of the system differential equation. For design purposes, modifications are imposed to the parameters of the system in order to obtain the desired response, and most important to the subject discussed hereon, this procedure can be automated.

Basically, two approaches can be used to perform the dynamic analysis required in this case: the *direct* and the *modal* method. The first one is devoted to situations in which only a few degrees of freedom are considered: calculations at discrete excitation frequencies are carried out by the use of complex algebra to obtain the solution of a set of coupled equations. For systems with a large number of degrees of freedom, on the other hand, the modal method utilizes the modal basis obtained for the homogeneous case to write the system response as a linear combination of the individual modal responses (Lalanne, 1983). In cases where proportional damping can be assumed to hold, the modal matrix is able to decouple the equations of motion, greatly simplifying the computations. However, in the case of general damping, state space representation is required to obtain the decoupled equations of motion (Meirovitch, 1997). In either case, it is important to mention the approximate nature of the modal method, implying that the accuracy of the results depends on the retention of a reasonable number of modes within the truncated modal matrix.

4. Application: exhaust system dynamic optimization (Butkewitsch, 1998)

After establishing the solution path to the structural synthesis of systems subjected to forced vibration, an illustrative example will follow according to the finite element model presented in Fig. 6. In this case study, the exhaust system of a heavy duty truck (muffler, pipes and supporting beams) is attached to the lower side of the vehicle' gearbox and is excited by a load whose frequency spectrum is broad enough to contain the five first resonant peaks of the structure. This situation leads to significant vibration displacement of the muffler and severe stresses capable of producing uncomfortable noise followed by fatigue failure of the supporting beams after a certain operation time.

This situation poses the need to design changes, and two complementary approaches were carried out.

(a) The first optimization problem (Butkewitsch and Steffen, 1998) consists of a classical natural frequencies repositioning procedure. The idea is to modify the structural configuration of the system to avoid resonances, which can be extremely severe for such a lightly damped vehicle assembly. The optimal design would lead, in the best case, to a new spectrum of natural frequencies avoiding coincidences with the gearbox excitation frequency. In the present case, a desirable situation for the steady state would be that of each excitation frequency being located between two resonant frequencies, but at a "safe distance" from them. Since this extremely ideal design is not possible in this particular problem due to the broad band excitation spectrum, a coincidence between natural and excitation frequencies should then occur a0t higher frequency values (mainly in the case of λ_1, the first resonant frequency) where the dynamical amplification effects are less severe. It is also desirable that any eventual gain in mass (ΔM) is not greater than 10%, relative to the original system (M). The mathematical translation of an optimization procedure intended to accomplish such a configuration can be stated as follows:

$$\max(\lambda_1) \iff \Delta M \leqslant 0.1M. \tag{13}$$

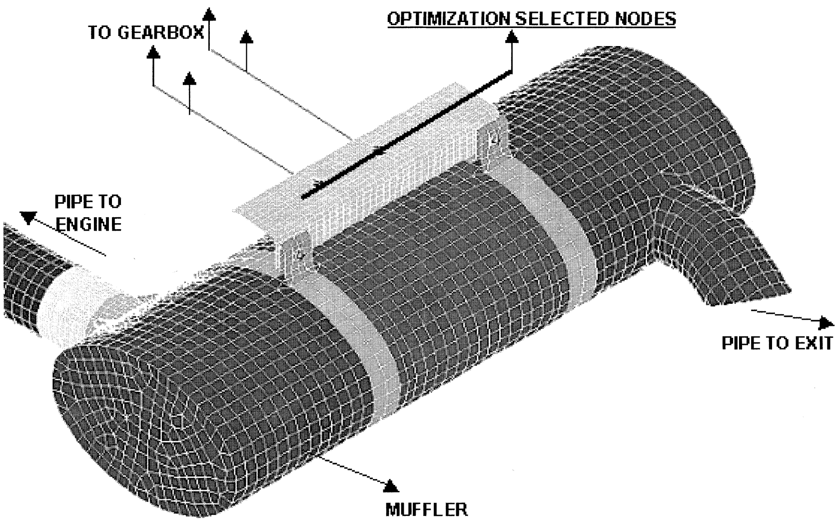

Fig. 6. Truck exhaust system subject to forced vibration.

The solution was implemented in a way to take advantage of the automation resources offered by the computational package employed: several conventional structural profiles were proposed to replace the supporting bars of the exhaust system shown in Fig. 6, and simultaneously, their principal dimensions were designed to find optimal values.

To do so, a library of the aforementioned software was used. This set contains the geometry of various structural profiles and the pre-defined design variables automatically associated with their defining dimensions. This implies that the engineer has, at his or her fingertips, several computational models for testing purposes without any additional pre-processing effort other than building the initial finite element model of the system.

Therefore, the software used to develop this work has, in addition to the resources needed to implement automated structural synthesis procedures, a high level of user interface automation, ensured by the flexibility offered in dimensional optimization procedures, in which the design variables do not need to coincide with the dimensions defined in the FEM model. The profiles tested in this case study, with the design variables automatically associated to them, are presented in Table 1.

Table 1
Tested dimensional optimization profiles and respective associated design variables

No.	Profile type	Design variables
1	Rectangular tube	*B*: Baselength *H*: Height *T*1: Horizontal wall thickness *T*2: Vertical wall thickness
2	"L"-profile	*L*1: Horizontal arm length *L*2: Vertical arm length *T*1: Horizontal arm thickness *T*2: Vertical arm thickness
3	Spar	*Ac*: Area of massive ends *H*: Distance between massive ends *T*: Rod thickness
4	Massive rectangular	*B*: Baselength *H*: Height
5	Rail	*B*1: Lower arm length *T*1: Lower arm thickness *B*2: Upper arm length *T*2: Upper arm thickness *H*: Profile height *T*3: Central rod thickness
6	Tube	*D*: External diameter *T*: Wall thickness
7	"I"-profile	*B*: Horizontal arm length *T*1: Horizontal arm thickness *H*: Vertical arm length *T*2: Vertical arm thickness
8	"T"-profile	*B*: Horizontal arm length *T*1: Horizontal arm thickness *H*: Vertical arm length *T*2: Vertical arm thickness

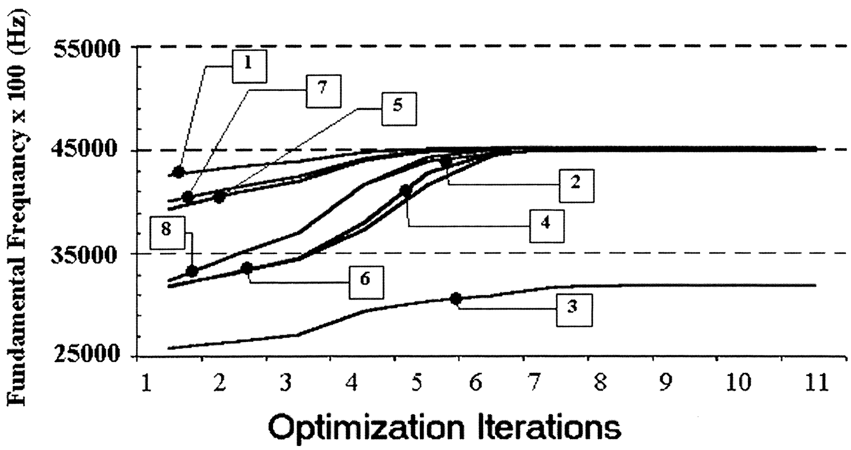

Fig. 7. Summary of first (modal) approach results.

A graphical representation of the results obtained with all tested structural profiles is shown in Fig. 7. From a dynamical point of view, the level of the objective function variation (i.e., the raise of the fundamental frequency) was highly satisfactory since the constraints were not violated in any case.

Regarding the optimization problem itself, it was possible, due to the ease of testing several different initial designs, to get strong numerical evidence that the optimum was obtained for the given design space since almost all the tested profiles (exception made to No. 3, the "Spar" profile) lead to very similar values for the first natural frequency.

(b) The second optimization procedure defined to find a better design for this structure is

- *Objective function*: minimize the maximum absolute value of the nodal displacements in key positions (indicated in Fig. 6 as "optimization selected nodes");
- *Design variables*: area properties of the supporting bar transverse sections;
- *Constraints*: side constraints on the values of the design variables, in order to fulfill assembly requirements.

Contrary to the previous approach, a forced vibration analysis is conducted within this new formulation of the optimization problem. Experimental data consisting of power spectra functions is supplied in order to establish which are the excitation features at key points of the structure and the proper kind of FEM simulations can be carried out to execute the optimization procedure.

In this case, data structure features of the optimization software call for some additional considerations that are of vital importance. Besides that the solution of the equation of motion is well established and yields no particular problem for the finite element solver itself, there is a limitation regarding the representation of dynamic physical quantities by the optimizer. This occurs because such quantities can assume several different values depending on the analysis circumstances, that is, the important entities related to the behaviour of the dynamic system (displacement, velocity and acceleration) have one different value for each forcing frequency per node. Thus, they cannot be defined neither as objective nor as constraints because such entities are represented by the optimizer as scalars (one should treat them as vectors in order to keep the information belonging to all analysed frequencies).

One reasonable alternative to this situation is to optimize the extreme value of the dynamic quantity. Very often, it is of practical interest to retain the maximum dynamically induced displacement between certain boundaries, or even minimize it down to the lowest possible value. Thus, the optimization problem

Table 2
Results of dynamic behaviour optimization for the heavy duty truck exhaust system

	Artificial objective "Beta" (non-dimensional, but proportional to nodal displacements)
Initial design	1.000000
Final design	4.137149×10^{-4}

could be formulated only in terms of the maximum displacement instead of regarding the displacements in all forcing frequencies, since it is not possible, as stated above.

Computationally, the maximum displacement (as well as the maximum value of any other entity) can be directly selected by the max function, an operator that returns only the maximum value displayed by a function (or a set of points contained in a vector, in the discrete case) over its domain. For optimization purposes, however, it may not be a suitable option because the max function is strongly discontinuous, which prompts a serious difficulty for a major part of the available optimization methods. So, instead of using the max function, the maximum value of a certain entity can be selected and minimized according to the algorithms presented by the equations below (VMA Engineering, 1996):

$$\begin{aligned} &\min(\text{Beta}), \quad 0 \leqslant \text{Beta} \leqslant 1, \\ &F(\text{Beta}, X) = \text{Beta} - X/Y \geqslant 0. \end{aligned} \tag{14}$$

The first of the equations above (which are the complete formulation of the "Beta method") shows the optimization problem expressed in terms of the auxiliary design variable "Beta", defined in the range between 0 and 1 for better numerical conditioning. The initial value for this variable is chosen depending on the aim of the optimization procedure: one, if the objective function is to be minimized and zero otherwise. The second equation is a constraint imposed over the difference of Beta and the real target of the optimization (in this case, the displacement X adjusted by the scale factor Y in order to have an order of magnitude similar to Beta). To minimize Beta and simultaneously respect the constraint, the value of X must decrease, and thus, its maximum is minimized indirectly.

An additional consideration ensures the proper functioning of the "Beta" method: the correct choice for Y's value. If it is too small, the constraint expressed by the second part of Eq. (14) is prone to be violated, which is undesirable. On the other hand, very big values lead to low sensitivities of X with respect to the design variables because the result of X/Y becomes too small in comparison with Beta (and only Beta is minimized with no concrete physical improvement). All in all, an adequate solution is setting the value of the scale factor Y in a way that the constraint $\text{Beta} - X/Y \geqslant 0$ almost but not violated. The results obtained in the case study presented in this paper are shown in Table 2 (just the value of Beta is displayed in order to protect proprietary rights contained in actual displacement data).

The corresponding variation obtained for the real optimization target (dynamical displacement, in this case) depends on the value of the scale factor Y adopted for the implementation of the "Beta" method.

5. Conclusions

The goal of this paper is to present a report covering the application of state-of-the-art techniques to real world industrial design situations. Automated structural synthesis of complex engineering systems represents an important challenge for modern industry and competitivity reasons justify the efforts to research and implement the methodology presented.

Both modal and forced vibration approaches led to very similar optimal design variable values and structural responses, indicating that the best possible solution was obtained for this case study within the limits imposed by the design task itself. If, for manufacturability reasons, the optimal solution cannot be implemented, another configuration for the exhaust system should be proposed and optimized. The presented "Beta Method" offers the possibility for a number of automated structural synthesis in frequency dependent situations. The runtimes are affordable for all cases studied, taking account of the fact that several finite element dynamic analysis were carried out. It should be added here that the model reduction algorithms bridging analysis and optimization play a decisive role in runtime reductions.

Finally, it can be mentioned that the methodology presented in this paper can be extended to a large class of design situations found in the industry.

Acknowledgements

The authors are thankful to Mr. Marcos Antônio Argentino and Mr. Udo Ricardo Wildmann of HCAE (Humaitá Cálculos Estruturais – Debis Humaitá IT Services, Latin America) for their supportive participation during the development of this work, providing experimental data, computer models and operational insight for the case study presented.

References

Blakely, K., 1993. Basic dynamic analysis user's guide, MSC/NASTRAN version 68, User's guide, The McNeal-Schwendler Corporation, Los Angeles, CA,USA.

Butkewitsch, S., 1998. Otimização de Sistemas Mecânicos: Ferramenta de Engenharia de Concepção, Masters degree dissertation, Federal University of Uberlândia.

Butkewitsch, S., Steffen Jr., V., 1998. Advanced mechanical design through automated structural synthesis. Ciência e Engenharia, Ano 7, Número 2, ISSN 0103-944X.

Canfield, R.A., 1990. High-quality approximation of eigenvalues in structural optimization. AIAA Journal 28 (6), 1116–1122.

Lalanne, M., et al., 1983. Mechanical Vibrations for Engineers. Wiley, Belfast.

Meirovitch, L., 1997. Principles and Techniques of Vibrations. Prentice-Hall, Engelwood Cliffs, NJ.

Moore, G., 1992. Design sensitivity and optimization, MSC/NASTRAN version 68, User's guide, The McNeal-Schwendler Corporation, Los Angeles, CA, USA.

Vanderplaats, G.N., 1998. Numerical optimization tools for engineering: with applications, VR&D, Springs, CO, USA.

Vanderplaats, G.N., Salajegheh, E., 1989. A new approximation method for stress constraints in structural synthesis. AIAA Journal 27 (3), 352–358.

VMA Engineering, 1996. VMA/GENESIS version 3.0 User's Guide, Vols. I, II and III, Colorado Springs, CO., USA.

Sergio Butkewitsch obtained his degree in Mechanical Engineering from Federal University of Uberlândia, Brazil, in 1995 and his Masters degree in the same field from the same university in 1998. At present, he is a Ph.D. student at the same institute. His research interest is in product and process statistical modelling, analysis and optimization.

Valder Steffen Jr. obtained his degree in Mechanical Engineering from UNICAMP, Brazil, in 1976 and his Ph.D. degree in 1979 and his Habilitation diploma in 1991, both from Besarçon University, France. He is a professor of Mechanical Engineering at the Federal University of Uberlândia, Uberlândia, MG, Brazil, and his research interest is in the dynamics of mechanical systems and optimization.